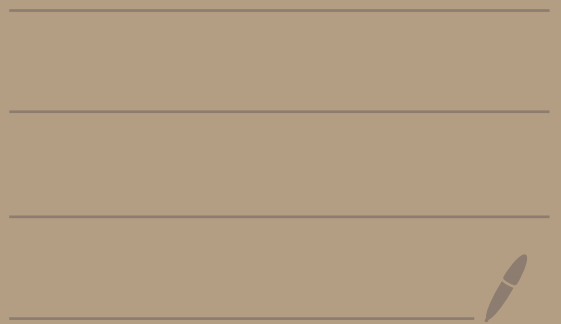


# Topic 2 - Matrices

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## HW 2

# MATRICES

①

Def: A matrix is a rectangular array of numbers. If  $M$  is a matrix and it has  $m$  rows and  $n$  columns then we say that  $M$  is an  $m \times n$  matrix.  
read: "m by n"

Abstractly we can write an  $m \times n$  matrix like this:

$$M = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

where  $a_{ij}$  is the entry in the  $i$ -th row and  $j$ -th column.

②

Ex:

$$M = \begin{pmatrix} 1 & 5 \\ 3 & -2 \end{pmatrix} = \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$$

$$a_{11} = 1$$

$$a_{12} = 5$$

$$a_{21} = 3$$

$$a_{22} = -2$$

M is a  $2 \times 2$  matrix.

Ex:

③

$$M = (5 \quad 0 \quad 10 \quad 7)$$

$$(a_{11} \quad a_{12} \quad a_{13} \quad a_{14})$$

M is a  $1 \times 4$  matrix.

$$a_{11} = 5$$

$$a_{12} = 0$$

$$a_{13} = 10$$

$$a_{14} = 7$$

Note: You can use commas if you want to make it clearer. Like this:

$$M = (5, 0, 10, 7)$$

Note: Sometimes we want to think of a vector as a matrix.

Suppose we have  $\vec{v} = \langle a_1, a_2, \dots, a_n \rangle$  in  $\mathbb{R}^n$ .

We can think of  $\vec{v}$  as an  $n \times 1$  matrix  $\begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{pmatrix}$

or we can think of  $\vec{v}$  as a  $1 \times n$  matrix  $(a_1 \ a_2 \ \dots \ a_n)$

Ex:  $\vec{v} = \langle 1, 5, \frac{1}{2} \rangle$

Can think of  $\vec{v}$  as

$$\begin{pmatrix} 1 \\ 5 \\ 1/2 \end{pmatrix}$$

3x1 matrix

or  $(1 \ 5 \ \frac{1}{2})$ .

1x3 matrix

Def: Let  $A$  and  $B$  be

(5)

$m \times n$  matrices.

[They have the same size.]

Let

$$A = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix}$$

and

$$B = \begin{pmatrix} b_{11} & b_{12} & \dots & b_{1n} \\ b_{21} & b_{22} & \dots & b_{2n} \\ \vdots & \vdots & & \vdots \\ b_{m1} & b_{m2} & \dots & b_{mn} \end{pmatrix}$$

(1) We define  $A+B$  to be the following  $m \times n$  matrix:

$$A+B = \begin{pmatrix} a_{11}+b_{11} & a_{12}+b_{12} & \dots & a_{1n}+b_{1n} \\ a_{21}+b_{21} & a_{22}+b_{22} & \dots & a_{2n}+b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1}+b_{m1} & a_{m2}+b_{m2} & \dots & a_{mn}+b_{mn} \end{pmatrix}$$

⑥

② We define  $A - B$  to be the following  $m \times n$  matrix:

$$A - B = \begin{pmatrix} a_{11} - b_{11} & a_{12} - b_{12} & \dots & a_{1n} - b_{1n} \\ a_{21} - b_{21} & a_{22} - b_{22} & \dots & a_{2n} - b_{2n} \\ \vdots & \vdots & & \vdots \\ a_{m1} - b_{m1} & a_{m2} - b_{m2} & \dots & a_{mn} - b_{mn} \end{pmatrix}$$

③ If  $\alpha$  is in  $\mathbb{R}$ , the scalar product  $\alpha A$  is defined to be the  $m \times n$  matrix:

$$\alpha A = \begin{pmatrix} \alpha a_{11} & \alpha a_{12} & \dots & \alpha a_{1n} \\ \alpha a_{21} & \alpha a_{22} & \dots & \alpha a_{2n} \\ \vdots & \vdots & & \vdots \\ \alpha a_{m1} & \alpha a_{m2} & \dots & \alpha a_{mn} \end{pmatrix}$$

Ex:

$$\begin{pmatrix} 0 & 5 \\ 3 & 1 \end{pmatrix} + \begin{pmatrix} 2 & -1 \\ 6 & 7 \end{pmatrix} = \begin{pmatrix} 0+2 & 5-1 \\ 3+6 & 1+7 \end{pmatrix}$$

$\underbrace{\hspace{10em}}_{2 \times 2}$        $\underbrace{\hspace{10em}}_{2 \times 2}$        $= \begin{pmatrix} 2 & 4 \\ 9 & 8 \end{pmatrix}$

Ex:

$$\begin{pmatrix} 1 \\ -1 \\ 1 \\ 1 \end{pmatrix} - \begin{pmatrix} 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} = \begin{pmatrix} 1-2 \\ -1-3 \\ 1-4 \\ 1-5 \end{pmatrix} = \begin{pmatrix} -1 \\ -2 \\ -3 \\ -4 \end{pmatrix}$$

$\underbrace{\hspace{10em}}_{4 \times 1}$        $\underbrace{\hspace{10em}}_{4 \times 1}$

Ex:

$$\begin{pmatrix} 1 & 5 \\ 3 & 2 \end{pmatrix} + \begin{pmatrix} 1 & 3 \\ 5 & 7 \\ 1 & 8 \end{pmatrix}$$

$\underbrace{\hspace{10em}}_{2 \times 2}$        $\underbrace{\hspace{10em}}_{3 \times 2}$

this sum is undefined since the matrices don't have the same size.



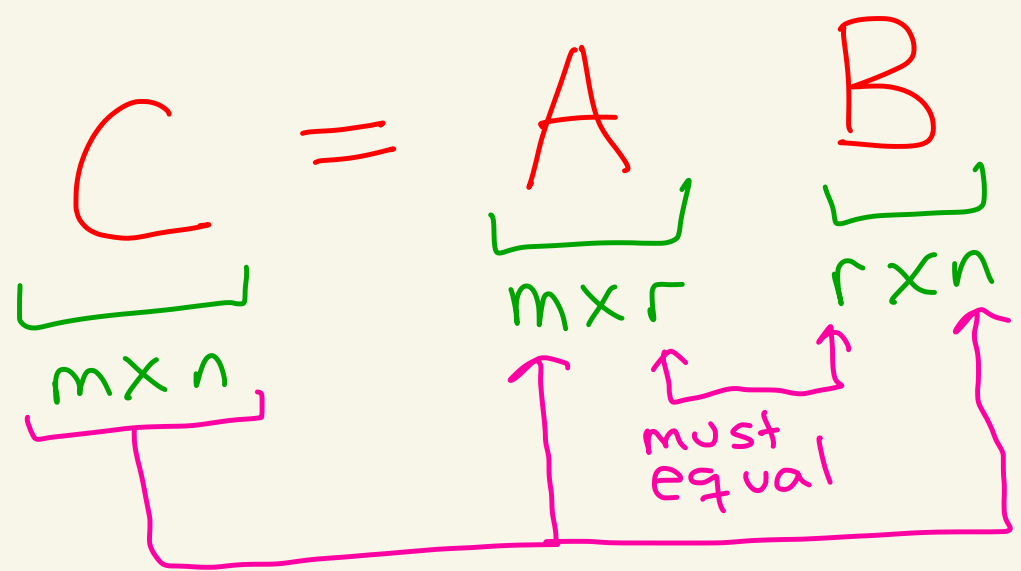
Ex:

8

$$10 \begin{pmatrix} 1 & 3 \\ -1 & 5 \end{pmatrix} = \begin{pmatrix} 10 & 30 \\ -10 & 50 \end{pmatrix}$$

---

Def: Let  $A$  be an  $m \times r$  matrix and  $B$  be an  $r \times n$  matrix. We define the product of  $A$  and  $B$ , denoted by  $AB$ , as the  $m \times n$  matrix  $C$  whose entry at row  $i$  and column  $j$  is defined to be the dot product of the  $i$ -th row of  $A$  and the  $j$ -th column of  $B$ .



Ex: Calculate AB, if possible,

Where

$A = \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$  and  $B = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix}$

Dimensions:  $2 \times 2$  for A,  $2 \times 3$  for B.

equal, so AB is defined

AB will be  $2 \times 3$

$AB = \begin{pmatrix} \text{(row 1 of A)} \cdot \text{(column 1 of B)} & \text{(row 1 of A)} \cdot \text{(column 2 of B)} & \text{(row 1 of A)} \cdot \text{(column 3 of B)} \\ \text{(row 2 of A)} \cdot \text{(column 1 of B)} & \text{(row 2 of A)} \cdot \text{(column 2 of B)} & \text{(row 2 of A)} \cdot \text{(column 3 of B)} \end{pmatrix}$

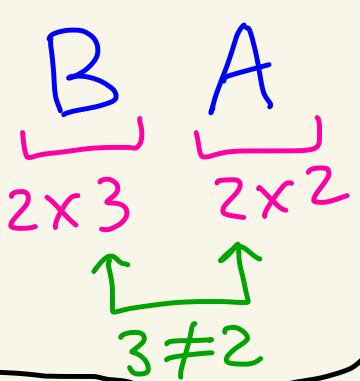
$$\begin{pmatrix} (1 \ 2) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} & (1 \ 2) \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} & (1 \ 2) \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} \\ (-1 \ 0) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} & (-1 \ 0) \cdot \begin{pmatrix} 2 \\ 1 \end{pmatrix} & (-1 \ 0) \cdot \begin{pmatrix} -1 \\ 0 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} (1)(1) + (2)(0) & (1)(2) + (2)(1) & (1)(-1) + (2)(0) \\ (-1)(1) + (0)(0) & (-1)(2) + (0)(1) & (-1)(-1) + (0)(0) \end{pmatrix} \quad \textcircled{11}$$

$$= \begin{pmatrix} 1 & 4 & -1 \\ -1 & -2 & 1 \end{pmatrix}.$$

AB

Ex: Using the same matrices  
can we calculate BA?



Since  $3 \neq 2$ , BA  
is not defined.

You can also see this if you  
tried to multiply them.

$$BA = \begin{pmatrix} 1 & 2 & -1 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -1 & 0 \end{pmatrix}$$

(row 1 of B) · (column 1 of A)

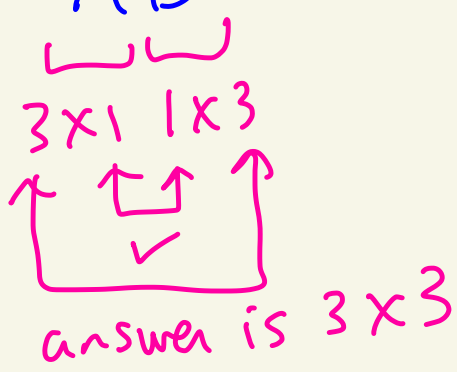
$$= \left( \begin{array}{l} (1 \ 2 \ -1) \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} \\ \text{-----} \\ \text{-----} \end{array} \right)$$

You can't do this dot product  
since the sizes aren't the same.

Ex: Let

$$A = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \text{ and } B = \begin{pmatrix} 0 & 1 & -3 \end{pmatrix}.$$

Calculate  $AB$  if possible.



$$AB = \begin{pmatrix} \text{(row 1 of A).} \\ \text{(column 1 of B)} \\ (1)(0) & \text{(row 1 of A).} \\ \text{(column 2 of B)} \\ (1)(1) & \text{(row 1 of A).} \\ \text{(column 3 of B)} \\ (1)(-3) \\ \text{(row 2 of A).} \\ \text{(column 1 of B)} \\ (2)(0) & \text{(row 2 of A).} \\ \text{(column 2 of B)} \\ (2)(1) & \text{(row 2 of A).} \\ \text{(column 3 of B)} \\ (2)(-3) \\ \text{(row 3 of A).} \\ \text{(column 1 of B)} \\ (-1)(0) & \text{(row 3 of A).} \\ \text{(column 2 of B)} \\ (-1)(1) & \text{(row 3 of A).} \\ \text{(column 3 of B)} \\ (-1)(-3) \end{pmatrix}$$

$$= \begin{pmatrix} 0 & 1 & -3 \\ 0 & 2 & -6 \\ 0 & -1 & 3 \end{pmatrix}$$

Ex: Let

$$A = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix} \text{ and } B = (0 \ 1 \ -3)$$

as before. Can we calculate BA?

$$BA = (0 \ 1 \ -3) \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

$1 \times 3$        $3 \times 1$   
 ↑      ✓      ↑      ↑  
 BA is  $1 \times 1$

(row 1)  
of B)  
(column 1)  
of A)

$$= (0 \ 1 \ -3) \cdot \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$$

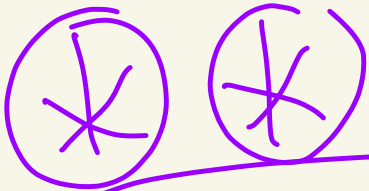
$$= ((0)(1) + (1)(2) + (-3)(-1))$$

$$= (5) \quad \leftarrow \text{BA is } 1 \times 1$$

Note:

In the previous examples  
when  $A = \begin{pmatrix} 1 \\ 2 \\ -1 \end{pmatrix}$  and  $B = (0 \ 1 \ -3)$

we saw that  $AB \neq BA$



In general,  $AB = BA$   
is NOT always true  
for matrices



Def: Let  $A$  be an  $m \times n$  matrix.

The transpose of  $A$ , denoted by  $A^T$ , is defined to be the  $n \times m$  matrix that results from interchanging the rows of columns of  $A$ .

That is, the  $i$ -th column of  $A^T$  is the  $i$ -th row of  $A$ .

[Similarly, the  $j$ -th row of  $A^T$  is the  $j$ -th column of  $A$ .]

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Some people write  $A^t$  instead of  $A^T$

Ex: Let

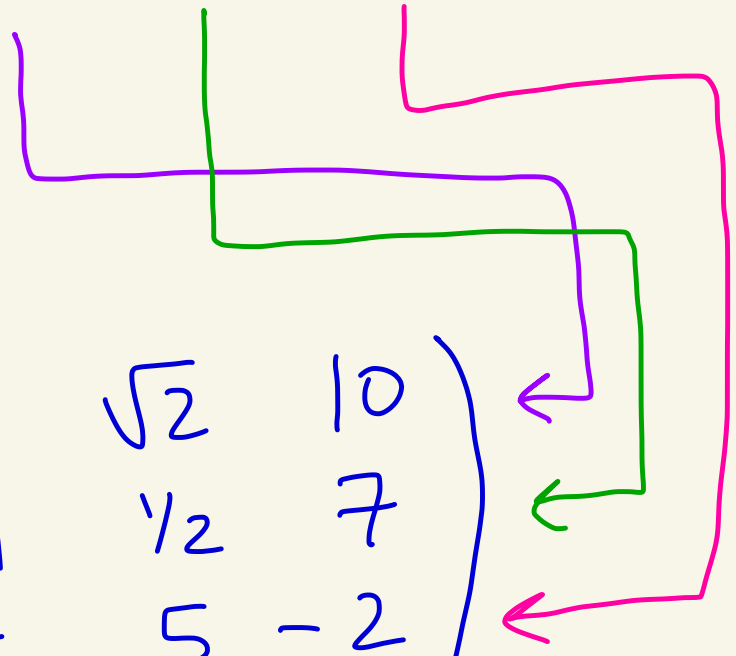
$$A = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{pmatrix} \quad 3 \times 4$$

$$A^T = \begin{pmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{pmatrix} \quad 4 \times 3$$

You could also have turned the columns of A into the rows of  $A^T$  and you'd get the same answer

Ex:

$$A = \begin{pmatrix} 1 & -1 & \pi \\ \sqrt{2} & \frac{1}{2} & 5 \\ 10 & 7 & -2 \end{pmatrix} \quad 3 \times 3$$

$$A^T = \begin{pmatrix} 1 & \sqrt{2} & 10 \\ -1 & \frac{1}{2} & 7 \\ \pi & 5 & -2 \end{pmatrix} \quad 3 \times 3$$


The diagram illustrates the transpose operation. Three colored lines connect corresponding elements between the two matrices: a purple line connects the element at row 1, column 1 of A to the element at row 1, column 1 of A^T; a green line connects the element at row 2, column 1 of A to the element at row 1, column 2 of A^T; and a pink line connects the element at row 3, column 1 of A to the element at row 1, column 3 of A^T. Similar connections exist for the other rows and columns.

Def: The  $m \times n$  zero matrix is the  $m \times n$  matrix where every entry is zero. We denote it by  $O_{m \times n}$  or just by  $O$  if we don't want to mention the size.

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Ex:

$$O_{2 \times 2} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$O_{4 \times 1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$O_{5 \times 3} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

Ex:

$$\text{Let } A = \begin{pmatrix} 1 & 5 \\ 7 & 2 \\ 3 & -1 \end{pmatrix}$$

(20)

Then,

$$A + O_{3 \times 2} = \begin{pmatrix} 1 & 5 \\ 7 & 2 \\ 3 & -1 \end{pmatrix} + \begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 5 \\ 7 & 2 \\ 3 & -1 \end{pmatrix} = A$$

Similarly,

$$O_{3 \times 2} + A = A$$

Def: The  $n \times n$  identity  
matrix, denoted by  $I_n$

[or just  $I$  when we don't want  
to or need to say the size],

is the  $n \times n$  matrix with 1's  
along the main diagonal and  
0's everywhere else.

Ex:

$$I_1 = (1)$$

$$I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$I_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$I_4 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

and so on.

Ex: Let  $A = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$  ←  $2 \times 2$  (23)

Consider

$$I = I_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

We have that

$$A I = \underbrace{\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}}_{2 \times 2} \underbrace{\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}}_{2 \times 2}$$

equal ✓

answer is  $2 \times 2$

$$= \begin{pmatrix} (1 \ 2) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} & (1 \ 2) \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ (3 \ 4) \cdot \begin{pmatrix} 1 \\ 0 \end{pmatrix} & (3 \ 4) \cdot \begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix}$$



$$= \begin{pmatrix} (1)(1) + (2)(0) & (1)(0) + (2)(1) \\ (3)(1) + (4)(0) & (3)(0) + (4)(1) \end{pmatrix} \quad (24)$$

$$= \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} = A$$

So,  $A I_2 = A$ .

You can also calculate

$I_2 A$  which is defined

$$\underbrace{I_2}_{2 \times 2} \underbrace{A}_{2 \times 2}$$

and you will get  $I_2 A = A$ .

Ex:

Let  $A = \begin{pmatrix} 1 & 3 & \pi \\ -1 & 2 & -2 \end{pmatrix}$   $\leftarrow$   $2 \times 3$

Note that

$$\underbrace{I_2}_{2 \times 2} \underbrace{A}_{2 \times 3} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 3 & \pi \\ -1 & 2 & -2 \end{pmatrix}$$

$\underbrace{\hspace{10em}}_{\checkmark}$   
 answer =  $2 \times 3$

$$= \begin{pmatrix} (1 \ 0) \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} & (1 \ 0) \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} & (1 \ 0) \cdot \begin{pmatrix} \pi \\ -2 \end{pmatrix} \\ (0 \ 1) \cdot \begin{pmatrix} 1 \\ -1 \end{pmatrix} & (0 \ 1) \cdot \begin{pmatrix} 3 \\ 2 \end{pmatrix} & (0 \ 1) \cdot \begin{pmatrix} \pi \\ -2 \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 3 & \pi \\ -1 & 2 & -2 \end{pmatrix} = A$$

Note that  $\underbrace{A}_{2 \times 3} \underbrace{I_2}_{2 \times 2}$  is not defined. (26)

But if you calculate

$$\underbrace{A}_{2 \times 3} \underbrace{I_3}_{3 \times 3} = \begin{pmatrix} 1 & 3 & \pi \\ -1 & 2 & -2 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
$$= \begin{pmatrix} 1 & 3 & \pi \\ -1 & 2 & -2 \end{pmatrix} = A$$

you fill in this part

$$\text{So, } AI_3 = A.$$

Theorem:

Let  $A, B, C$  be matrices  
and let  $\alpha, \beta$  be real numbers.

$\alpha = \text{alpha}$   
 $\beta = \text{beta}$

Then the following are  
true where we will

assume that the sizes of the  
matrices are such that the  
operations are defined:

- ①  $A + B = B + A$
- ②  $A + (B + C) = (A + B) + C$
- ③  $A(BC) = (AB)C$
- ④  $A(B + C) = AB + AC$
- ⑤  $(B + C)A = BA + CA$
- ⑥  $A(B - C) = AB - AC$

7)  $(B - C)A = BA - CA$

8)  $\alpha(B + C) = \alpha B + \alpha C$

9)  $\alpha(B - C) = \alpha B - \alpha C$

10)  $(\alpha + \beta)A = \alpha A + \beta A$

11)  $(\alpha - \beta)A = \alpha A - \beta A$

12)  $\alpha(\beta A) = (\alpha\beta)A$

13)  $\alpha(AB) = (\alpha A)B = A(\alpha B)$

14)  $(A^T)^T = A$

15)  $(A + B)^T = A^T + B^T$

16)  $(A - B)^T = A^T - B^T$

17)  $(\alpha A)^T = \alpha A^T$

18)  $(AB)^T = B^T A^T$

note the reversal of the order

(19) If  $A$  is  $m \times n$ , then (29)

$$A I_n = A.$$

(20) If  $A$  is  $m \times n$ , then

$$I_m A = A$$

(21) If  $A$  is  $m \times n$ , then

$$A - A = O_{m \times n}$$

(22) If  $A$  is  $m \times n$ , then

$$A + O_{m \times n} = O_{m \times n} + A = A$$

Lets prove part (5) of the previous theorem for 2x2 matrices

[This is HW 2-Part 2 #1(a)]

Suppose that  $A, B, C$  are  $2 \times 2$  matrices. } Given

Prove that  $(B+C)A = BA + CA$  } what you need to show

proof:

Let  $A, B, C$  be  $2 \times 2$  matrices. } declare what the objects are

Then,  
 $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, B = \begin{pmatrix} e & f \\ g & h \end{pmatrix}, C = \begin{pmatrix} i & j \\ k & l \end{pmatrix}$   
where  $a, b, c, d, e, f, g, h, i, j, k, l$  are real numbers.

Then,

$$(B+C)A = \left[ \begin{pmatrix} e & f \\ g & h \end{pmatrix} + \begin{pmatrix} i & j \\ k & l \end{pmatrix} \right] \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

do first because of parentheses

$$= \begin{pmatrix} e+i & f+j \\ g+k & h+l \end{pmatrix} \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

$\underbrace{\hspace{10em}}_{2 \times 2} \quad \underbrace{\hspace{10em}}_{2 \times 2}$   
 answer is  $2 \times 2$

$$= \begin{pmatrix} (e+i) \cdot \begin{pmatrix} a \\ c \end{pmatrix} & (f+j) \cdot \begin{pmatrix} b \\ d \end{pmatrix} \\ (g+k) \cdot \begin{pmatrix} a \\ c \end{pmatrix} & (h+l) \cdot \begin{pmatrix} b \\ d \end{pmatrix} \end{pmatrix}$$

$$= \begin{pmatrix} (e+i)a + (f+j)c & (e+i)b + (f+j)d \\ (g+k)a + (h+l)c & (g+k)b + (h+l)d \end{pmatrix}$$




$$= \begin{pmatrix} ea+ia+fc+jc & eb+ib+fd+jd \\ ga+ka+hc+lc & gb+kb+hd+ld \end{pmatrix} (*)$$

We also have that

$$BA+CA = \underbrace{\begin{pmatrix} e & f \\ g & h \end{pmatrix}}_{2 \times 2} \underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{2 \times 2} + \underbrace{\begin{pmatrix} i & j \\ k & l \end{pmatrix}}_{2 \times 2} \underbrace{\begin{pmatrix} a & b \\ c & d \end{pmatrix}}_{2 \times 2}$$

answer is 2x2

$$= \begin{pmatrix} (e \ f) \cdot \begin{pmatrix} a \\ c \end{pmatrix} & (e \ f) \cdot \begin{pmatrix} b \\ d \end{pmatrix} \\ (g \ h) \cdot \begin{pmatrix} a \\ c \end{pmatrix} & (g \ h) \cdot \begin{pmatrix} b \\ d \end{pmatrix} \end{pmatrix}$$

$$+ \begin{pmatrix} (i \ j) \cdot \begin{pmatrix} a \\ c \end{pmatrix} & (i \ j) \cdot \begin{pmatrix} b \\ d \end{pmatrix} \\ (k \ l) \cdot \begin{pmatrix} a \\ c \end{pmatrix} & (k \ l) \cdot \begin{pmatrix} b \\ d \end{pmatrix} \end{pmatrix} =$$


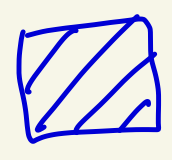
$$= \begin{pmatrix} ea+fc & eb+fd \\ ga+hc & gb+hd \end{pmatrix} + \begin{pmatrix} ia+jc & ib+jd \\ ka+lc & kb+ld \end{pmatrix}$$

$$= \begin{pmatrix} ea+fc+ia+jc & eb+fd+ib+jd \\ ga+hc+ka+lc & gb+hd+kb+ld \end{pmatrix} (**)$$

We can see that (\*) equals (\*\*).

Thus,

$$(B+C)A = BA + CA.$$



other ways:  
QED,  $\square$

$\leftarrow$  (end of proof symbol)

Let's prove part (15) for 3x2 matrices.  
[This is HW 2 - Part 2 #1 (F)]

Let A and B  
be 3x2 matrices.

} Given

Prove that  
 $(A+B)^T = A^T + B^T$

} Need to show

Proof:

Let A and B be  
3x2 matrices.

} declare the objects

Then,  
 $A = \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}$  and  $B = \begin{pmatrix} g & h \\ i & j \\ k & l \end{pmatrix}$

where a, b, c, d, e, f, g, h, i, j, k, l  
are real numbers.

We have that

$$(A+B)^T = \left( \left( \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix} + \begin{pmatrix} g & h \\ i & j \\ k & l \end{pmatrix} \right)^T$$

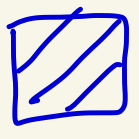
add first because of parentheses

$$= \begin{pmatrix} a+g & b+h \\ c+i & d+j \\ e+k & f+l \end{pmatrix}^T = \begin{pmatrix} a+g & c+i & e+k \\ b+h & d+j & f+l \end{pmatrix}$$

$$= \begin{pmatrix} a & c & e \\ b & d & f \end{pmatrix} + \begin{pmatrix} g & i & k \\ h & j & l \end{pmatrix}$$

$$= \begin{pmatrix} a & b \\ c & d \\ e & f \end{pmatrix}^T + \begin{pmatrix} g & h \\ i & j \\ k & l \end{pmatrix}^T$$

$$= A^T + B^T$$



# HW 2 - Part 2

(2)(a) Suppose that  $A, B, C, D$  are  $n \times n$  matrices. Use the properties from class to show that

$$(A+B)(C+D) = AC + AD + BC + BD$$

proof: Let  $A, B, C, D$  be  $n \times n$  matrices. Then,

$$(A+B)(C+D) = (A+B)C + (A+B)D$$

*(Note: Brackets in the original image indicate dimensions:  $(A+B)$  is  $n \times n$ ,  $C$  is  $n \times n$ ,  $(A+B)D$  is  $n \times n$ , and  $D$  is  $n \times n$ . The word "ok" is written above the brackets.)*

$X(C+D) = XC + XD$   
Property (4) from class  
Use  $X = A+B$

$$= AC + BC + AD + BD$$

prop. (1)  
 $X+Y = Y+X$

$$= AC + AD + BC + BD$$

$(M+N)X = MX + NX$   
Property (5) from class

